Photometry Transformation Coefficients

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This note is a summary of techniques discussed and derived by Henden and Kaitchuck [2] and generalized to standard combinations of filters as recommended by the AAVSO in a document by Gary [1]. In the following sections we use capital letters, e.g. A, to represent a standard filter bandpass magnitude. Lower case letters, e.g. a, represent the instrument filter bandpass magnitude after it is transformed to a value above the atmosphere. (Transformation to above-atmosphere values requires the determination and application of extinction coefficients as outlined in §2.)

1 Measuring and using transformation coefficients in differential photometry

Color transformation coefficients are based on two neighboring bands, call them a and b. By convention, we take the wavelengths associated a and b, λ_a and λ_b, to be such that $\lambda_a < \lambda_b$. Standard pairings of a and b are: r and i; v and r; b and v; u and b. A set of neighboring pairs of bands, in a multi-band measurement, are used to define the first order color coefficients C_{ab} and T_{ab}. Using capital letters to represent transformed magnitudes and lower case to represent measured (instrumental) magnitudes corrected to above-atmosphere values (see §2). The color transformation coefficients are defined by

$$A - B = C_{ab} + T_{ab}(a - b). \tag{1}$$

Magnitude transformation coefficients C_a and T_a also take into account the effect of color and are defined by

$$A - a = C_a + T_a(A - B) \tag{2}$$

or, if the color to the shorter wavelength side is prefered

$$B - b = C_b + T_b(A - B). \tag{3}$$

To measure the transformation coefficients we need stars of known A and B (e.g. in a Landolt field) and some instrumental magnitude measurements a and b of those stars corrected to above-atmosphere values, using extinction coefficients (see §2) with known air mass. Find the transformation coefficients through least squares fits to

$$A - a = C_a + T_a(A - B) \tag{4}$$

(or $B - b = C_b + T_b(A - B)$) and

$$a - b = C_0 + C_1(A - B) \tag{5}$$

where it will follow that

$$C_{ab} = -C_0/C_1 \text{ and } T_{ab} = 1/C_1. \tag{6}$$

In the following subsections we let A_o and a_o be the transformed and above-atmosphere instrumental magnitudes, respectively, of the object star. Let A_c and a_c be the transformed and above-atmosphere instrumental magnitudes, respectively, of the comparison star. The definition of the transformation coefficients to use in each case are as given by Gary [1]. The UBV case is as outlined in Henden and Kaitchuck [2]. The perscriptions "slope of y versus x" mean y as ordinate, x as abcissa; y is always a measured quantity ("dependent variable"), while x is known ("independent variable").

1.1 BVRI case

For this case we need the following transformation coefficients (and the zero intercepts for all-sky work), with measured values (dependent variable) on the ordinate:

- T_r = slope of $(R - r)$ versus $(R - I)$
- T_v = slope of $(V - v)$ versus $(V - R)$
- T_{ri} = reciprocal of the slope of $(r - i)$ versus $(R - I)$
- T_{vr} = reciprocal of the slope of $(v - r)$ versus $(V - R)$
- T_{bv} = reciprocal of the slope of $(b - v)$ versus $(B - V)$

For differential photometry use (in order):

$$(R_o - I_o) = (R_c - I_c) + T_{ri}[(r_o - i_o) - (r_c - i_c)] \tag{7}$$
$$R_o = r_o + (R_c - r_c) + T_r[(R_o - I_o) - (R_c - I_c)] \tag{8}$$
$$I_o = R_o - (R_o - I_o) \tag{9}$$
$$V_o = v_o + (V_c - v_c) + T_v T_{vr}[(v_o - r_o) - (v_c - r_c)] \tag{10}$$
$$B_o = V_o + (B_c - V_c) + T_{bv}[(b_o - v_o) - (b_c - v_c)] \tag{11}$$

1.2 VRI case

For this case we need the following transformation coefficients (and the zero intercepts for all-sky work), with measured values (dependent variable) on the ordinate:

- T_r = slope of $(R - r)$ versus $(R - I)$
- T_v = slope of $(V - v)$ versus $(V - R)$
- T_{ri} = reciprocal of the slope of $(r - i)$ versus $(R - I)$

- T_{vr} = reciprocal of the slope of $(v - r)$ versus $(V - R)$

For differential photometry use (in order):

$$\begin{aligned}
(R_o - I_o) &= (R_c - I_c) + T_{ri}[(r_o - i_o) - (r_c - i_c)] && (12)\\
R_o &= r_o + (R_c - r_c) + T_r[(R_o - I_o) - (R_c - I_c)] && (13)\\
I_o &= R_o - (R_o - I_o) && (14)\\
V_o &= v_o + (V_c - v_c) + T_v T_{vr}[(v_o - r_o) - (v_c - r_c)] && (15)
\end{aligned}$$

1.3 BVR case

For this case we need the following transformation coefficients (and the zero intercepts for all-sky work), with measured values (dependent variable) on the ordinate:

- T_v = slope of $(V - v)$ versus $(V - R)$
- T_{vr} = reciprocal of the slope of $(v - r)$ versus $(V - R)$
- T_{bv} = reciprocal of the slope of $(b - v)$ versus $(B - V)$

For differential photometry use (in order):

$$\begin{aligned}
(V_o - R_o) &= (V_c - R_c) + T_{vr}[(v_o - r_o) - (v_c - r_c)] && (16)\\
V_o &= v_o + (V_c - v_c) + T_v[(V_o - R_o) - (V_c - R_c)] && (17)\\
R_o &= V_o - (V_o - R_o) && (18)\\
B_o &= V_o + (B_c - V_c) + T_{bv}[(b_o - v_o) - (b_c - v_c)] && (19)
\end{aligned}$$

1.4 BVI case

For this case we need the following transformation coefficients (and the zero intercepts for all-sky work), with measured values (dependent variable) on the ordinate:

- T_v = slope of $(V - v)$ versus $(V - I)$
- T_{vi} = reciprocal of the slope of $(v - i)$ versus $(V - I)$
- T_{bv} = reciprocal of the slope of $(b - v)$ versus $(B - V)$

For differential photometry use (in order):

$$\begin{aligned}
(V_o - I_o) &= (V_c - I_c) + T_{vi}[(v_o - i_o) - (v_c - i_c)] && (20)\\
V_o &= v_o + (V_c - v_c) + T_v[(V_o - I_o) - (V_c - I_c)] && (21)\\
I_o &= V_o - (V_o - I_o) && (22)\\
B_o &= V_o + (B_c - V_c) + T_{bv}[(b_o - v_o) - (b_c - v_c)] && (23)
\end{aligned}$$

1.5 RI case

For this case we need the following transformation coefficients (and the zero intercepts for all-sky work), with measured values (dependent variable) on the ordinate:

- T_r = slope of $(R - r)$ versus $(R - I)$
- T_{ri} = reciprocal of the slope of $(r - i)$ versus $(R - I)$

For differential photometry use (in order):

$$\begin{aligned} (R_o - I_o) &= (R_c - I_c) + T_{ri}[(r_o - i_o) - (r_c - i_c)] && (24) \\ R_o &= r_o + (R_c - r_c) + T_r[(R_o - I_o) - (R_c - I_c)] && (25) \\ I_o &= R_o - (R_o - I_o) && (26) \end{aligned}$$

1.6 VR case

For this case we need the following transformation coefficients (and the zero intercepts for all-sky work), with measured values (dependent variable) on the ordinate:

- T_v = slope of $(V - v)$ versus $(V - R)$
- T_{vr} = reciprocal of the slope of $(v - r)$ versus $(V - R)$

For differential photometry use (in order):

$$\begin{aligned} (V_o - R_o) &= (V_c - R_c) + T_{vr}[(v_o - r_o) - (v_c - r_c)] && (27) \\ V_o &= v_o + (V_c - v_c) + T_v[(V_o - R_o) - (V_c - R_c)] && (28) \\ R_o &= V_o - (V_o - R_o) && (29) \end{aligned}$$

1.7 VI case

For this case we need the following transformation coefficients (and the zero intercepts for all-sky work), with measured values (dependent variable) on the ordinate:

- T_v = slope of $(V - v)$ versus $(V - I)$
- T_{vi} = reciprocal of the slope of $(v - i)$ versus $(V - I)$

For differential photometry use (in order):

$$\begin{aligned} (V_o - I_o) &= (V_c - I_c) + T_{vi}[(v_o - i_o) - (v_c - i_c)] && (30) \\ V_o &= v_o + (V_c - v_c) + T_v[(V_o - I_o) - (V_c - I_c)] && (31) \\ I_o &= V_o - (V_o - I_o) && (32) \end{aligned}$$

1.8 BV case

For this case we need the following transformation coefficients (and the zero intercepts for all-sky work), with measured values (dependent variable) on the ordinate:

- T_b = slope of $(B - b)$ versus $(B - V)$
- T_{bv} = reciprocal of the slope of $(b - v)$ versus $(B - V)$

For differential photometry use (in order):

$$(B_o - V_o) = (B_c - V_c) + T_{bv}[(b_o - v_o) - (b_c - v_c)] \quad (33)$$
$$B_o = b_o + (B_c - b_c) + T_b[(B_o - V_o) - (B_c - V_c)] \quad (34)$$
$$V_o = B_o - (B_o - V_o) \quad (35)$$

1.9 UBV case

For this case we need the following transformation coefficients (and the zero intercepts for all-sky work), with measured values (dependent variable) on the ordinate:

- T_v = slope of $(V - v)$ versus $(B - V) = \epsilon$ in [2] (note the sign reversal for T_v over the definition given in Equation (4) because $\lambda_B < \lambda_V$)
- T_{bv} = reciprocal of the slope of $(b - v)$ versus $(B - V) = \mu$ in [2]
- T_{ub} = reciprocal of the slope of $(u - b)$ versus $(U - B) = \psi$ in [2]

For differential photometry use (in order):

$$(B_o - V_o) = (B_c - V_c) + T_{bv}[(b_o - v_o) - (b_c - v_c)] \quad (36)$$
$$V_o = v_o + (V_c - v_c) + T_v[(B_o - V_o) - (B_c - V_c)] \quad (37)$$
$$(U_o - B_o) = (U_c - B_c) + T_{ub}[(u_o - b_o) - (u_c - b_c)] \quad (38)$$

1.10 Same field and different field differential photometry

To use the transformations of this section for differential photometry, technically the above-atmosphere instrumental magnitudes are used. When the object star and comparison star are in different fields, as when a calibrated comparison star is being defined in a variable star field, the above-atmosphere instrumental magnitudes need to be used to account for the different air masses for the two fields. When the comparison star and object star are nearby in the same CCD field (essentially the same air mass for each star) then using the instrumental magnitudes directly in the equations of this section will result in higher precision. This is because extinction corrections can add systematic error that is greater than the correction if the air mass difference between the object and comparison star is small. However, uncorrected same field photometry should only be done at low airmass (above roughly 20° altitude to avoid the effects of differential color extinction with changing airmass.

2 Extinction coefficients

For all-sky photometry the instrumental magnitudes a need to be transformed to above-atmosphere (zero air mass) instrumental magnitudes a_0 using

$$a_0 = a - (k'_a + k''_a c)X \tag{39}$$

where k'_a and k''_a are the first and second order extinction coefficients, respectively, X is the airmass and c is an instrumental color index like $b - a$ (it should be the difference between a and a neighboring band instrumental magnitude b with $\lambda_b < \lambda_a$ if possible, otherwise use $c = a - b$ with $\lambda_a < \lambda_b$). The air mass is approximated in the flat Earth model by

$$X = \sec z \tag{40}$$

where z is the zenith angle (90° - altitude). Instrumental color indicies $c = a - b$ can be transformed to an above-atmosphere value $c_0 = (a - b)_0$ by

$$c_0 = c - (k'_c + k''_c c)X. \tag{41}$$

Note that $k'_c = k'_a - k'_b$ and $k''_c = k''_a - k''_b$. In the UBV system, $k''_{ub} \equiv 0$ (so $k''_u = k''_b$) and k''_v is known to be very small.

2.1 Measuring extinction coefficients

Dropping the second order coefficients from Equations (39) and (41) leaves:

$$a = a_0 + k'_a X \tag{42}$$

and

$$c = c_0 + k'_c X \tag{43}$$

so that the first order extinction coefficients (and the above-atmosphere values) can be found from a least squares fit of the measured instrumental magnitude versus air mass X. More exactly, Equations (39) and (41) rearranged are

$$a = a_0 + (k'_a + k''_a c)X \tag{44}$$

and

$$c = c_0 + (k'_c + k''_c c)X \tag{45}$$

so the slope of the least squares fits are $(k'_a + k''_a c)$ and $(k'_c + k''_c c)$.

The second order extinction coefficients can be found from close pairs of stars (e.g. red-blue pairs) by measuring the instrumental magnitudes a and b of each to get Δa and Δc. Since X is essentially the same for both stars we have

$$\Delta a = \Delta a_0 + k''_a (\Delta c)X \tag{46}$$

and the second order extinction coefficient k_a is found from the slope of a least squares slope of measured instrumental magnitude differences Δa versus $(\Delta c)X$ where Δc is the measured instrumental color index difference.

With extinction coefficients in hand, all measured instrumental magnitudes may be converted to above-atmosphere values and the transformation coefficients of §1 may be measured.

2.2 A note on differential photometry

Equations (39) and (41) can be rearranged to give the above-atmosphere instrumental magnitude a_0 from the measured instrumental magnitude a as

$$a_0 = a - k'_a X - k''_a c X \tag{47}$$

and the above-atmosphere instrumental color index c_0 from the measured instrumental color index c as

$$c_0 = (1 - k''_c X)c - k'_c X. \tag{48}$$

So if two stars, 1 and 2 (e.g. object and comparison star), are in the same field with essentially the same X then the differences in their above-atmosphere magnitudes and color indicies are

$$(a_1 - a_2)_0 = (a_1 - a_2) + k''_a (c_1 - c_2) X \tag{49}$$

and

$$(c_1 - c_2)_0 = (1 - k''_c X)(c_1 - c_2). \tag{50}$$

These are the quantities that should be used in the equations of §1 for same-field differential photometry. Note that if the object and comparison stars are similar in color, then the air mass correction can be small enough to be negelected and instrumental magnitudes used directly in the equations of §1. If you are working at high airmass, the second order extinction coefficient is needed.

References

[1] Gary BL, "CCD Transformation Equations for use with Single-Image Photometry", URL: `http://reductionism.net.seanic.net/CCD_TE/cte.html` (2006)

[2] Henden AA, Kaitchuck, RH, 1982, *Astronomical Photometry*, Van Nostrand Reinhold Company (1982)